Quadratic surface fitting to locate the center of circular feature

# Design of algorithm:

The algorithm finds coefficients of a quadratic surface model with a cost function, Eq. (1), over a local patch, Ω.

(1)

The critical point, , is obtained by partial derivatives of Eq. (1). The solution is at where the partial derivatives become zero,

(2)

For a numerical stability, the process requires to interpolate the local patch Ω from an initial guess, . The initial guess, an approximate location of the critical point, is from other low-level image processing technique such as the center of gravity in connected component.

Once critical point is found from Eq. (2), it serves as new initial guess, and the algorithm iterates by re-establishing Eq. (1) with the updated . This iterative process continues until the critical point is converged. The algorithm does not update result if the correction is greater than threshold.

In the implementation, the method generates a number of blurred regions with different smoothing parameters to cope with varying individual feature size. The smoothing and interpolation processes help better fit the local patch, Ω, of the input image to the quadratic model.

Note that the implementation provides solution efficiently by estimating parameters using pseudo inverse (instead of least square technique) in iterative process. Pseudo inverse of a nominal window center at (0, 0), , is used while the interpolation is performed in the original coordinate frame. Then, sought parameters are to find a relative position adjustment is obtained as in Eq. (3).

(3)

is computed only once.